

TRB Paper No. 01-0205

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TITLE: OPTIMIZATION OF TRANSFER COORDINATION FOR INTERMODAL TRANSIT NETWORKS

by

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Transportation Research Board
80th Annual Meeting
January 7-11, 2001
Washington, D.C.

OPTIMIZATION OF TRANSFER COORDINATION FOR INTERMODAL TRANSIT NETWORKS

ABSTRACT

In most urban areas where transit demand spreads widely, passengers may be served by an intermodal transit system, consisting of a rail transit line (or a bus rapid transit route) and different numbers of feeder routes connecting at different transfer stations. Schedule synchronization may significantly reduce transfer times at stations where various routes are interconnected. Since vehicles arrive stochastically, slack time reserved in schedules may be desirable to increase the probability of transfer connections. The developed objective total cost function, including supplier and user costs, is minimized by finding the optimal headways and slack times for coordinated routes. A two-stage procedure is developed and applied into an example, while the optimal train-bus coordinated groups at transfer stations are found.

Keywords: Transit, Intermodal, Transfer, Coordination, Optimization, Planning

ACKNOWLEDGEMENTS

This work was partially supported (under no contract) by the National Center for Transportation and Industrial Productivity (NCTIP) and the Interdisciplinary Program in Transportation at the New Jersey Institute of Technology.

1. INTRODUCTION

In 1991, Intermodal Surface Transportation Efficiency Act (ISTEA) emphasize on developing intermodal system, which reflects in its policy statement “*to develop a National Intermodal Transportation System that is economically sound, provides the foundation for the nation to compete in the global economy and will move people and goods in an energy efficient manner*” [US Department of Transportation, 1991]. The successor of ISTEA, Transportation Equity Act for the 21st century (TEA-21), further supports and encourages the development of intermodal system. Transfer time is one of the most important service quality indicators for evaluating intermodal transit systems, which should be minimized with available technology, such as transfer coordination. Short headway, substantially reduce transfer time, is not always cost effective due to the variation of demand over space and time. Thus efficient transfer coordination among transit routes may increase the attractiveness to passengers and productivity of the system. Due to stochastic headway variation, synchronization of vehicle arrivals may not be cost effective. Slack time reserved into schedule is a valuable way to increase the probability of transfer connections.

In this study, an objective total cost function including supplier and user costs is developed for optimizing transfer coordination within a linear intermodal transit system, as shown in Figure 1. The intermodal system includes a rail transit line and numbers of feeder bus routes connecting at stations along the train line. A two-stage procedure is also developed for determining the optimal train-bus coordinated groups at transfer stations.

2. LITERATURE REVIEW

In the design of optimal transit operations, a common objective is to minimize the total cost, including operator and user costs [Hurdle, 1973; Wirasinghe, Hurdle and Newell, 1977; Kuah and Pert, 1988; Lee and Schonfeld, 1991; Chien and Schonfeld 1997; Chien and Chowdhury 1997; Chowdhury and Chien, 2000; Chowdhury, 2000]. Some of previous studies [Hurdle, 1973; Kuah and Pert, 1988; Wirasinghe, 1980; Chien and Schonfeld, 1997] formulated mathematical models for a train system served with feeder buses. However, none of them considered coordination for a train line and its feeder buses.

Hurdle [1973] formulated a total cost function (including supplier and user costs), while the optimal locations of parallel feeder bus lines and headways were found. Wirasinghe [1980] optimized feeder bus route locations, bus headways and train station locations through minimizing the similar total cost function. Later, Kuah and Pert [1988] developed an analytical model for a feeder bus system. The decision variables, such as route spacing, operating headway and stop spacing are jointly optimized.

Chien and Schonfeld [1997] conducted a similar study, but considered more decision variables, such as bus stop spacings. They formulated a model to jointly optimize the service provided by a train line and its feeder bus system. Considering the demand and parameters distributed irregularly along the service area, the train line length, train station spacing, train

headway, bus headways, bus route and stop spacings with and without train-bus coordination were optimized. In that model, both train and bus headways were assumed deterministic.

Theoretical studies suggested that effective schedule coordination may significantly increase the attractiveness and productivity of linear intermodal transit system [Hicky, 1992]. However, few studies [Lee and Schonfeld, 1991; Chien and Chowdhury, 1997; Chowdhury and Chien, 2000; Chowdhury 2000] attempted to optimize schedule coordination for such system. Lee and Schonfeld [1991] formulated a model to determine optimal vehicle slack times for a transfer terminal serving multiple bus routes. Stochastic vehicle arrivals were considered while formulating the objective coordinated transfer cost function. Analytical results were derived for both empirical and gumbel distributions of vehicle arrival times. They found that the standard deviation of vehicle arrival times is an important factor affecting the durations of slack times.

Later, Chowdhury and Chien [2000] formulated a model for coordinating an intermodal transit system, while normal bus arrival time distributions were assumed. They found that coordination is preferable if the train and feeder bus headways are large.

3. ASSUMPTIONS

In order to formulate the total cost function for optimizing the transfer coordination within the intermodal transit network, the following assumptions are made.

- (1) Demand is independent with the quality of transit service (i.e., fixed demand). The demand pattern of feeder bus routes is many-to-one, whose distribution is uniform over space and a given time period.
- (2) Locations of transit facilities (e.g., routes and stations), parameters on supply side (e.g., vehicle sizes, operating speeds and cost) and demand side (e.g., value of user's time, demand density and distribution) of the analyzed system are given.
- (3) Administration, vehicle maintenance, insurance, labor, and energy consumption costs are factors of vehicle operation cost. The train operation cost is fixed in this study by assuming the train size (number of cars per train) is a constant. In real world operation, train size can be adjustable for service demand in different time periods. This assumption can be relaxed by setting that the train operating cost is the cost for operating one car train multiplied by the train size, which has been discussed by Chien and Schonfeld (1998).
- (4) Bus arrivals, unlike the train line with exclusive right of way and deterministic headway, are stochastic. A common headway is designed at a station for coordinated operation.
- (5) The probability of the late vehicle arrival time beyond its headway is small enough to be negligible.

4. DEMAND FUNCTIONS

Transfer demand is defined as the number of passengers transferring from one route to another at transfer stations, which can be classified into bus-to-bus, bus-to-train and train-to-

bus transfer demand. All feeder bus routes serving demand toward and outward transfer stations are denoted as directions 1 and 2, respectively. If δ represents the bus services direction, $I_{ij\delta}$ means the hourly demands of bus route j at transfer station i and can be obtained from Eq. (1):

$$I_{ij\delta} = \begin{cases} \sum_{d=1}^2 U_{ijd} + \sum_{k=1}^{m_i} U_{ijk} & \text{for } \delta = 1, \quad 1 \leq i \leq n, 1 \leq j \leq m_i \\ \sum_{d=1}^2 U_{idj} + \sum_{k=1}^{m_i} U_{ikj} & \text{for } \delta = 2, \quad 1 \leq i \leq n, 1 \leq j \leq m_i \end{cases} \quad (1)$$

where d represents the train service direction ($d=1$: from station 1 to n , and $d=2$: from station n to 1). In Eq. 1, U_{ijd} and U_{idj} represent the transfer demand from bus route j at station i to train direction d and vice versa, respectively. Similarly, U_{ijk} is the transfer demand from bus route j to route k , where k varies from 1 the number of feeder bus routes connecting at station i , called m_i .

The bi-directional train inflow and outflow at each station are derived and shown in Eqs. 2 and 3, respectively. Note that train stations are not necessary connecting with feeder service when the demand is low. In general, if train station i is served by feeder bus routes, the inflow demand of train direction d , called I_{id} , is the sum of demands from all feeder routes at the station to train direction d plus the demand α_{id} who walk or take transportation modes other than buses to the station.

$$I_{id} = \sum_{j=1}^{m_i} U_{ijd} + \alpha_{id} \quad \text{for } d \in \{1,2\}, 1 \leq i \leq n \quad (2)$$

$$Q_{id} = \sum_{j=1}^{m_i} U_{idj} + \beta_{id} \quad \text{for } d \in \{1,2\}, 1 \leq i \leq n \quad (3)$$

where Q_{id} and β_{id} are the train outflow and demand destining at station i from train direction d , respectively.

5. TOTAL COST FUNCTION

The objective function is defined as the total system cost (C_T), including supplier cost C_O and user cost C_U , is formulated in this section for optimizing the transfer coordination within the intermodal transit network. The supplier cost is incurred by operating trains and buses in the system, while the user cost includes wait, transfer, and in-vehicle costs. Since the demand is fixed and the locations of transit routes and stations are given, the user access cost is a constant and will not affect the optimization results, and thus can be omitted in the

objective function. All variables used to formulate the total cost function are defined in Table 1. The derivation of each cost component is discussed below.

Supplier Cost (C_o)

The supplier cost is the sum of train and bus operating costs. Both operating costs are formulated based on average round trip time, in which the layover times are assumed constant and thus will not affect the optimization results. Generally, the operating cost is defined as the fleet size multiplied by the train/bus operating cost (u_r for trains or u_b for buses), where the fleet size can be obtained from the round trip time divided by the headway (H_r for train or H_{ij} for bus route j at station i).

Considering the coordination of bus arrivals, it may be cost-effective to add slack time K_{ij} into the schedule of bus route j at station i to increase the probability of successful connection. Hence, the cost caused by the slack time should be considered as a portion of travel time. Thus, the supplier cost incurred by trains and buses is

$$C_o = \frac{T_r^R u_r}{H_r} + \sum_{i=1}^n \sum_{j=1}^{m_i} \frac{(T_{ij}^R + K_{ij}) u_b}{H_{ij}} \quad (4)$$

In Eq. 4, the first and the second terms represent the train and bus operator costs, where K_{ij} is 0 if route j at station i is not a coordinated route. The train and bus round trip times, called T_r^R and T_{ij}^R , constituted by moving and dwell times can be formulated as

$$T_{ij}^R = 2 \left(\frac{L_{ij}}{S_{ij}} + \frac{\sum_{\delta=1}^2 I_{ij\delta} H_{ij}}{q_b} \right) \quad \text{for } 1 \leq i \leq n, 1 \leq j \leq m_i \quad (5)$$

$$T_r^R = \frac{2L_r}{S_r} + \frac{\sum_{i=1}^n \sum_{d=1}^2 (I_{id} + Q_{id}) H_r}{q_r} \quad (6)$$

where q_b and q_r represent bus and train passengers boarding/alighting rates, respectively.

User Cost (C_U)

The total user cost defined here is the sum of wait cost, transfer cost and in-vehicle cost, incurred by passengers waiting for initial boarding, transferring from one route to another, and travelling in-vehicles. The details of these cost components are discussed below.

Wait Cost (C_w)

The wait cost, incurred by passengers waiting for buses or trains, can be obtained from the product of average wait time and demand multiplied by the value of users' wait time u_w . Where, average wait time depends on the factors including transit (train or bus) service headway, headway variation and passenger arrival distribution. For instance, if transit headway is completely regular and passenger arrivals are random, average wait time is one half of service headway. However, when the service headway and passenger arrivals are random, average wait time can be estimated based on the mean and variance of vehicle headways (Weldings, 1957). If both random and non random passenger arrival occurs, Turnquist (1978) developed a model to estimate average wait time, while taking the proportion of non-random arrivals, expected wait time for non-random and random arrivals as input of the model. For simplifying the model, in the numerical example average wait times for both trains and buses are assumed one half of the headway. The user wait cost is

$$C_w = \frac{1}{2} \left(\sum_{i=1}^n \sum_{j=1}^{m_i} H_{ij} I_{ij1} + \sum_{i=1}^n \sum_{d=1}^2 H_r \alpha_{id} \right) u_w \quad (9)$$

where I_{ij1} is the demand of bus route j in direction 1 at station i . The wait cost incurred by the transfer passengers from buses to the train and vice versa is considered in transfer cost.

Transfer Cost (C_t)

The transfer cost, incurred by transfer passengers from one route to another can be obtained by the product of average transfer time and transfer demand multiplied by the value of users' wait time. Thus, the total transfer cost C_t at all stations is formulated as

$$C_t = \sum_{i=1}^n \sum_{j=1}^{m_i} \sum_{k=1}^{m_i} [y_{ikj} T_{ikj}^C + (1 - y_{ikj}) T_{ikj}^N] U_{ikj} u_w + \sum_{i=1}^n \sum_{j=1}^{m_i} \sum_{d=1}^2 [y_{idj} T_{idj}^C + (1 - y_{idj}) T_{idj}^N] U_{idj} u_w \\ + \sum_{i=1}^n \sum_{d=1}^2 \sum_{j=1}^{m_i} [y_{idj} T_{ijd}^C + (1 - y_{idj}) T_{ijd}^N] U_{ijd} u_w \quad (10)$$

The transfer cost, including bus-to-bus, train-to-bus and bus-to-train transfer costs, is the sum of the first, second and third terms, in Eq. 10, respectively. U_{ikj} , U_{idj} , and U_{ijd} represent transfer demand from route k to j , train direction d to j and route j to train direction d at station i , respectively. The binary variable y_{idj} (e.g. 0 and 1) indicates the status of coordination between train direction d and bus route j at station i , while y_{ikj} represents the coordination between bus route k and j . Considering the benefit of coordination among routes, both coordinated and not coordinated transfer times for the transfers of bus-to-

bus (T_{ikj}^C and T_{ikj}^N), train-to-bus (T_{idj}^C and T_{idj}^N) and bus-to-train (T_{ijd}^C and T_{ijd}^N) are formulated and discussed next.

Unlike the average wait time, the average transfer times with and without coordination are formulated differently.

(1) Transfer Time without Coordination

Considering the randomness in both pickup and delivery vehicles arrival time at transfer stations, the bus-to-bus transfer time can be estimated by using Weldings [1957] model as shown in Eq. 5. Since train arrival at stations is deterministic, bus-to-train transfer time is one half of train headway.

(2) Transfer Time with Coordination

If both routes are coordinated, the average transfer time T_{ikj}^C induced by passengers transferring from route k (delivery vehicles) to j (pickup vehicles) can be obtained from the joint probability function of vehicles arrival time (Figure 2). The average transfer time depends on the slack time reserved in the schedule for dispatching the pickup vehicle and its arrival distributions. Therefore, transfer time accounts for times sensitive to slack time and headway. Which depends on the pickup and delivery vehicles arrival times in three ways as: (1) both pickup and delivery vehicles arrive on time (2) the pickup vehicle is late, and (3) the pickup vehicle left before the arrival of the delivery vehicle. In the first two situations coordinated transfer is successful, while, in the third situation, all passengers from the delivery vehicle will miss the coordinated pickup vehicle and wait for the next pickup one. Therefore, bus-to-bus transfer time consists of three components: (1) slack delay time, (2) connection delay time, and (3) missed connection delay time.

The connection delay time is the delay experienced by passengers when a transfer connection between two vehicles on coordinated routes is successfully made, but the pickup vehicle arrives behind schedule. It may exist only at the following two situations: (1) the delivery vehicle arrives before schedule and the pickup vehicle arrives after schedule, and (2) both the delivery and pickup vehicles arrive after schedule but the delivery one arrives before the pickup one. Thus, the connection delay time for passengers transferring from route k to route j at station i is

$$D_{ikj} = \int_{-H}^{K_k} f(t_{ik}) dt_{ik} \int_{K_j}^H (t_{ij} - K_{ij}) f(t_{ij}) dt_{ij} + \int_{K_k}^H f(t_{ik}) dt_{ik} \int_{t_{ik}-K_k+K_j}^H (t_{ij} - t_{ik} + K_{ik} - K_j) f(t_{ij}) dt \quad (11)$$

where i ranges from 1 to n, while j and k range between 1 and m_i .

The missed connection delay time is the delay experienced by passengers when connection between two coordinated vehicles are unsuccessful due to the departure of the pickup vehicle earlier than the arrival of the delivery vehicle. The missed connection delay

time may exist at two situations: (1) the pickup vehicle arrives before schedule and the delivery vehicle arrives after schedule and (2) both delivery and pickup vehicles arrive after schedule, but the delivery vehicle arrives after the departure of the pickup vehicle. In both situations, transfer passengers will miss the pickup vehicle and have to wait for the next pickup one. The average missed connection delay time for passengers transferring from route k to route j at station i is

$$M_{ikj} = \left[\int_{K_{ik}}^H f(t_{ik}) dt_{ik} \int_{-H}^{K_{ij}} f(t_{ij}) dt_{ij} + \int_{K_{ik}}^H f(t_{ik}) dt_{ik} \int_{K_{ij}}^{t_{ik}-K_{ik}+K_{ij}} f(t_{ij}) dt_{ij} \right] H \text{ for } 1 \leq i \leq n, 1 \leq j \leq m_i \quad (12)$$

Thus, the average transfer time T_{ikj}^C for passengers transferring from route k to j at station i is

$$T_{ikj}^C = K_{ik} + D_{ikj} + M_{ikj} \quad \text{for } 1 \leq i \leq n, 1 \leq j \leq m_i, 1 \leq k \leq m_i \quad (13)$$

The train-to-bus and bus-to-train transfer times considering coordinated and not coordinated situation can be similarly derived.

In-vehicle Cost (C_V)

In general, the in-vehicle cost is the product of the average in-vehicle time, the corresponding demand and the value of in-vehicle time. The in-vehicle cost is the summation of in-bus cost and in-train cost. The in-bus cost is formulated on the basis of average bus journey time, which accounts for the average run time and dwell time. The in-bus time, due to bus run time, is the average journey distance divided by the average bus operating speed (S_{ij}). Because of the uniform demand distribution over routes, the average journey distance is one half of the route length ($L_{ij}/2$). In addition, the average bus dwell time can be derived by the demand divided by boarding and alighting rate.

The in-vehicle cost is the sum of moving cost, while passengers travel between stations, and dwell cost, while trains dwell at stations. For example, the moving cost incurred by train passengers is the sum of inter-station travel times (which is interstation spacing l_i divided by average train operating speed V_r), multiplied by the number of passengers traveling between stations and the value of user's in-vehicle time. Similarly, the dwell cost is the number of in-train passengers multiplied by the dwell time and the value of user in-vehicle time. The trains dwell time at stations depends on the total number of boarding and alighting passengers. The dwell time is the sum of inflow and outflow demand at the station multiplied by the headway and divided by passenger boarding and alighting rate q_r . The in-vehicle cost, including in-bus and in-train cost, is formulated as

$$\begin{aligned}
C_v = & \sum_{i=1}^n \sum_{j=1}^{m_i} \sum_{\delta=1}^2 \left(\frac{L_{ij}}{2S_{ij}} + \frac{H_{ij}I_{ij\delta}}{2q_b} \right) I_{ij\delta} u_v + \sum_{i=1}^{n-1} \sum_{d=1}^2 \frac{A_{id} l_i u_v}{V_r} \\
& + \sum_{i=2}^{n-1} \sum_{d=1}^2 [A_{id} - \{I_{id}(2-d) + Q_{id}(d-1)\}] (I_{id} + Q_{id}) \frac{H_r}{q_r} u_v
\end{aligned} \tag{14}$$

where A_{i1} and A_{i2} represent $\sum_{l=1}^i (I_{l1} - Q_{l1})$ and $\sum_{m=i+1}^n (I_{m2} - Q_{m2})$, respectively.

After deriving all cost components, the total cost function C_T is expressed in Eq. 15, which will be minimized sequentially in a two-stage procedure discussed in the next section. The optimal decision variables (e.g., common headway and slack times) can be obtained subject to the capacity and non-negativity constraints formulated in Eqs. 16 and 17, respectively.

$$C_T(H_{ij}, H_r, K_{ij}) = C_O(H_{ij}, H_r, K_{ij}) + C_U(H_{ij}, H_r, K_{ij}) \text{ for } 1 \leq i \leq n, 1 \leq j \leq m_i \tag{15}$$

$$\text{Subject to: } \begin{cases} P_{ij} \geq D_{ij} \\ P_r \geq D_r \end{cases} \text{ for } 1 \leq i \leq n, 1 \leq j \leq m_i \tag{16}$$

$$\begin{cases} H_{ij} > 0 \\ H_r > 0 \\ K_{ij} \geq 0 \end{cases} \text{ for } 1 \leq i \leq n, 1 \leq j \leq m_i \tag{17}$$

where P_{ij}, P_r, D_{ij} , and D_r represent the service capacity of bus route j and train, and the maximum load of route j and train, respectively. The derivation of P_{ij} and P_r have been discussed by Chowdhury [2000].

6. OPTIMIZATION

The total cost formulated in Eq. 15 is minimized sequentially in a two-stage procedure discussed later, while the optimal decision variables in each stage are determined. At Stage I, the optimal headways of all routes are determined without coordination, while at Stage II, train-bus coordination at all transfer stations is considered. At Stage-I, headways can be optimized analytically. However, at stage-II, the optimal decision variables are difficult to be determined because of realistic vehicle arrival distributions (e.g., normal and lognormal distributions). Therefore, numerical integration (Gaussian Quadrature [Press, H. William et al. 1992]) is used to calculate the joint probability of pickup and delivery vehicles arrival under different situations. Powell's algorithm [Powell M. J. D. 1964], a multidimensional numerical search algorithm, which minimizes functions by performing successive one-dimensional search along a set of n linear independent and mutually conjugate directions from

an initial point, is used to optimize the decision variables (e.g., common headway and slack times).

Stage-I: No Coordination

At this Stage, since the coordination is not considered, the total cost derived in Eq. 15 is purely a function of route headways. After taking partial derivative of the total cost function with respect to each route headway and setting it equal to zero, the analytical relations between bus (or train) headways and other parameters (i.e., average operating cost, demand, value of users' time, etc) can be established. The uniqueness and the optimality of the headways are confirmed after checking the second derivative of cost function with respect to headway, which is positive. The total cost at stage-I can be obtained after substituting the optimized train and bus route headways into Eq. 15.

Stage-II: Train-Bus Coordination

Train-bus coordination involves identification of potential train directions to be coordinated with buses and the determination of coordinated bus groups at each transfer station. Considering the difficulty to synchronize train arrivals from both directions at each transfer station, the direction with higher transfer demand (sum of train-to-bus and bus-to-train demand) is considered to coordinate with buses. In order to find the optimal coordinated group at each transfer station an algorithm is developed, which iteratively evaluates various combinations of bus routes to be coordinated with trains. In the algorithm, bus routes, including all transfer stations, are ranked (listed) in a descending order based on the sum of demand transferring from bus-to-train and train-to-bus. At first iteration, all bus routes at all transfer stations are coordinated with train. Then for a new iteration, a bus route is removed from the list. The iteration continues until all bus routes are removed from the list. While a bus route is removed from the list, that route is no further coordinated at subsequent iterations.

To remove a bus route from the list, the following rule is applied. If in the previous iteration the optimal common headway is equal to the service capacity headway of any coordinated route, then that route is removed. Otherwise, the last route in the current list is removed. At each iteration, the optimal common headway and slack times of coordinated routes are obtained by minimizing the total cost function (see Eq. 15). After all iterations, the global minimum total cost can be identified, while the coordinated status of each connected route at the transfer station emerges.

The step by step procedure for optimizing coordinated operation in the intermodal system is discussed below.

- Step 1 List bus routes connected to all transfer stations in a descending order based on the sum of train-to-bus and bus-to-train demand.
- Step 2 Identify the coordinated train direction at each transfer station. [At each transfer station the train service direction with higher transfer demand (sum of train-to-bus and bus-to-train demand) will be coordinated with buses].
- Step 3 Coordinate all bus routes contain in the current list with train; optimize the decision variables (e.g., common headway and slack times of all coordinated bus routes) using Powell's algorithm, and calculate the total cost by substituting the decision variables into Eq. 15.
- Step 4 If optimized common headway is equal to service capacity headway of any coordinated bus route, remove that bus route from the list, otherwise, remove the last bus route from the list. If at least one route remains in the current list go to step 3, otherwise go to step 5.
- Step 5 Search the minimum total cost from the calculated total costs at step 3, while the coordinated status of each route can be obtained.
- Step 6 Compare the total cost identified in Step 5 and the total cost obtained from Stage I. Find the minimum total cost, while the coordinated status of each route can be obtained.

7.0 NUMERICAL EXAMPLE

The major purpose of this section is to optimize the coordination of a linear intermodal transit network by using the developed two-stage procedure. The results are obtained from a computer program coded in FORTRAN. The studied network, as shown in Figure 1, contains a 16-mile train line serving 11 stations. Stations 1, 2, 5, and 11 are transfer stations connecting with 6, 4, 4 and 5 feeder bus routes, respectively. Buses arrival times at transfer stations follow normal distributions [Taylor, 1982; Knoppers and Muller, 1995].

The baseline values of design variables and model parameters (e.g., value of users' wait time, value of operator cost, acceleration rate, deceleration rate, speed etc) are listed in Table 1. The vehicle capacities for buses operating on feeder routes and one-car trains operating on train line are assumed to be 80 pass/bus and 250 pass/car (including standees), respectively. The length of feeder routes and the standard deviation of vehicle arrival times at transfer stations from each bus route and demands are shown in Table 2, while the train station spacing and demands are shown in Table 3. By using the baseline values of design variables and model parameters, the decision variables (e.g., headways and slack times) for the analyzed intermodal transit network are optimized, which are discussed next.

Stage-I Optimization

At Stage-I, decision variables (i.e., bus and train headways) are optimized without considering coordination. The bus operator and user costs (i.e., wait, transfer, and in-vehicle costs) at each transfer station are obtained. The optimized bus and train headways and operator and user costs are shown in Table 4. At this stage, the minimum total costs is found to be 13119.23 \$/hr.

Stage-II Optimization

Following the Stage-II procedure, we found that the best transfer coordination for the intermodal transit network is that bus routes at stations 2 and 5 should be coordinated with train directions 1 and 2, respectively. In addition, bus routes at station 1 and 11 are desired to coordinate with both train directions. After ranking bus routes connected to all transfer stations in a descending order based on the sum of train-to-bus and bus-to-train demand, the total cost with train-bus coordination at various iterations are obtained and shown in Table 5, where the desired coordination is identified. The optimized common headway and coordinated bus routes subject to capacity constraints at each transfer station are also highlighted. The minimum total cost achieved in this stage is 12239.2 \$/hr, lower than that obtained from Stage-I.

Various cost components obtained from Stage-1 and -II optimization and the benefit of coordination in terms of costs savings are shown in Table 6. We found that the proposed transfer coordination model significantly reduces both the user and operator costs for the intermodal transit network. The wait and in-vehicle costs increase due to coordinated operation, but a larger saving from transfer cost is compensated. In this example, the overall benefit of transfer coordination is 880.02 \$/hr. The optimized headways and slack times are also shown in Table 6. The optimal slack times of coordinated routes vary from 0.035 to 0.063 hrs, depending on the variation of vehicle arrival times and the transfer demand. In general, the increasing variation of vehicle arrival times and transfer demand may cause the increase of slack time.

Sensitivity Analysis

Sensitivity analysis of the common headway of coordinated service at a transfer station with respect to total cost is presented, which confirms the relationship established in the model developed in Section 5. Figure 3 shows that the total cost is convex with respect to common headway. The optimal common headway is 0.345-hr that achieves the minimum total cost subject to the service capacity constraint.

Figure 4 shows how the standard deviation of vehicle arrival times affects the total cost both with and without coordination. The standard deviation of vehicle arrival time of routes #1 to #6 (coordinated group) at station 1 are varied from 0.001 to 0.065 hr, while the corresponding total cost with (Stage II) and without coordination (Stage I) are computed.

From the figure, it is shown that as the standard deviation of vehicle arrival time increases the benefit of coordination decreases.

8. CONCLUSION

The transfer coordination for an intermodal transit network consisting of train and feeder bus routes has been optimized in this study. Analytical approach is applied to optimize headways without coordination, while numerical search algorithm (the Powell's algorithm) is applied to find the optimal coordinated headways and slack times. The coordination optimization model developed in this study can be applied to a real intermodal transit network.

In the analyzed network, it is found that the train-bus coordination at all transfer station is desirable because of the low demand and long headway of train line. If the train demand is high, the optimal headway will decrease significantly to reduce the user cost. In such a situation, train-bus coordination at all transfer station may not be desirable.

Passenger demand is assumed to be inelastic in this study. A model with demand elasticity consideration can properly address fares policy or optimize system objectives that include consumer surplus. Therefore, the passenger demand should vary with the level of service provided by the intermodal transit system. The total cost minimization model developed in this study may be integrated with an elastic demand model to iteratively optimize the intermodal transit system coordination. A model that analytically integrates our supply system optimization with demand equilibrium approach, discussed by Kocur and Hendrickson (1995) and Chien and Spasovic (1999), would be a desirable extension.

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Table 1 Notation

C_O	=	Total supplier cost (\$/hr);
C_W	=	Total wait cost (\$/hr);
C_t	=	Total transfer cost (\$/hr);
C_V	=	Total in-vehicle cost (\$/hr);
C_U	=	Total user cost (\$/hr);
C_T	=	The total cost of intermodal transit system (\$/hr);
D_{ikj}	=	Connection delay time from route k to j at station i (hr);
d	=	Index of train service directions;
D_{ij}	=	Maximum bus load of route j at station i (pass./hr);
D_r	=	Maximum train load (pass.);
H_{ij}	=	Headway of bus route j at station I (hr);
H_r	=	Train headway (hr);
$I_{ij\delta}$	=	Demand of bus route j at station i in direction δ (pass/hr);
i	=	Index of train stations;
I_{id}	=	Train demand in direction 1 at station i (pass./hr);
j	=	Index of bus route;
k	=	Index of bus route;
K_{ij}	=	Slack time of bus route j at station i (hr);
L_{ij}	=	Length of bus route j at station i (miles);
L_r	=	Train line length (miles)
l_i	=	Interstation spacing between station i and i+1 (miles)
l	=	Index of train stations;
m	=	Index of train stations;
m_i	=	Number of bus routes at station i;
M_{ikj}	=	Missed connection delay time from route k to j (hr);
M_{ijd}	=	Missed connection delay time from route j to train direction d (hr);
n	=	Total number of train stations;
P_{ij}	=	Service capacity of buses of route j at station i (pass.);
P_r	=	Service capacity of train (pass.);
Q_{id}	=	Train outflows at station i in direction d (pass./hr);
q_b	=	Bus passengers boarding and alighting rate (1800 hr/pass.);
q_r	=	Train passengers boarding and alighting rate (21600 hr/pass.);
S_{ij}	=	Average bus operating speed of route j at station i (20 mph);
S_r	=	Average train operating speed (40 mph);

T_{ikj}^C	=	Transfer time from route k to j (under coordination) (hr);
T_{ikj}^N	=	Transfer time from route k to j (no coordination) (hr);
T_{ijd}^C	=	Transfer time from route j to train direction d (under coordination) (hr);
T_{ijd}^N	=	Transfer time from route j at station i to train dir. d (no coordination) (hr);
t_{ij}	=	Vehicle arrival time distribution of route j at station i (hr);
T_{ij}^R	=	Round trip time of bus route j at station i (hr);
T_r^R	=	Train round trip time (hr);
u_W	=	Value of passenger wait time (7 \$/pass-hr);
u_b	=	Average bus operating cost (80 \$/bus-hr);
u_r	=	Average train operating cost (180 \$/train-hr);
u_v	=	Value of user in-vehicle time (5 \$/hr);
U_{ijd}	=	Transfer demand from route j to train direction d at station i (pass/hr);
U_{ikj}	=	Transfer demand from route k to route j at station i (pass/hr);
U_{idj}	=	Transfer demand from train direction d to route j at station i (pass/hr);
y_{idj}	=	(0 or 1) indicating that train direction d and vehicle route j at transfer station i are (without or with)coordination;
y_{ikj}	=	(0 or 1) indicating that route k and j at transfer station i are (without of with) coordination;
α_{id}	=	train demand walk to station i in direction d (pass/hr);
β_{id}	=	Demand destining at station i from train direction d (pass/hr); and
δ	=	Bus route direction (1 or 2).

Table 2 Input for Bus Routes

$i^{(1)}$	$j^{(2)}$	BL^3	$STD^{(4)}$	$I_{ij1}^{(5)}$	$I_{ij2}^{(6)}$	Route to Route Demand Matrix (U_{ikj}) (pass/hr)						Buses to Train Demand (U_{ijd}) (pass/hr)	
						1	2	3	4	5	6	$d_1^{(7)}$	$d_2^{(8)}$
						1	1	7	0.035	130	111	0	43
	2	9	0.045	76	110	23	0	19	6	8	1	19	0
	3	9	0.05	80	81	26	26	0	7	9	1	11	0
	4	5	0.03	180	201	13	7	5	0	144	2	9	0
	5	5	0.02	220	196	22	13	9	165	0	4	7	0
	6	5	0.025	20	11	0	0	0	0	9	0	11	0
2	1	4	0.02	80	125	0	2	36	25	-	-	13	4
	2	9	0.035	25	31	4	0	2	2	-	-	12	5
	3	6	0.025	85	91	43	3	0	31	-	-	6	2
	4	7	0.025	79	74	36	4	31	0	-	-	6	2
5	1	7	0.02	210	172	0	21	13	126	-	-	17	33
	2	9	0.03	110	104	22	0	33	17	-	-	13	25
	3	10	0.04	70	91	14	25	0	11	-	-	7	13
	4	7	0.02	170	172	110	26	17	0	-	-	6	11
11	1	9	0.03	110	99	0	30	25	8	11	-	0	36
	2	10	0.025	95	98	24	0	19	8	10	-	0	34
	3	8	0.03	40	96	10	9	0	4	4	-	0	13
	4	4	0.025	250	237	17	15	10	0	200	-	0	8
	5	4	0.02	280	231	22	20	20	213	0	-	0	5

(1) = index of train station

(2) = index of bus route

(3) = bus route length (miles)

(4) = standard deviation of bus arrival time at transfer stations (hr)

(5) = demand of bus route j at station i in direction 1 (pass/hr)

(6) = demand of bus route j at station i in direction 2 (pass/hr)

(7)=train direction 1

(8)=train direction 2

Table 3 Input for the Train Line

Train Station Index (<i>i</i>)	$l_i^{(1)}$	Bus route (<i>j</i>)	$U_{ij}^{(2)}$	$U_{ij}^{(3)}$	$I_{i1}^{(4)}$	$I_{i2}^{(5)}$	$Q_{i1}^{(6)}$	$Q_{i2}^{(7)}$
1	2	1	0	27	90	0	0	172
		2	0	21				
		3	0	16				
		4	0	12				
		5	0	13				
		6	0	2				
2	1	1	28	14	50	18	70	37
		2	14	8				
		3	14	8				
		4	10	6				
3	1.5	-	-	-	5	6	7	10
4	2.5	-	-	-	6	5	6	9
5	1	1	14	12	65	115	55	55
		2	15	17				
		3	13	15				
		4	10	8				
6	1.5	-	-	-	9	10	8	6
7	2	-	-	-	6	7	5	3
8	1.5	-	-	-	7	6	3	2
9	2	-	-	-	12	8	4	5
10	1	-	-	-	7	10	7	6
11	-	1	26	0	0	120	92	0
		2	24	0				
		3	19	0				
		4	4	0				
		5	6	0				

(1) = spacing between station *i* and *i*+1 (miles)

(2) = transfer demand from train direction 1 to bus route *j* at station *i* (pass/hr)

(3) = transfer demand from train direction 2 to bus route *j* at station *i* (pass/hr)

(4) = train inflow at station *i* in direction 1 (pass/hr)

(5) = train inflow at station *i* in direction 2 (pass/hr)

(6) = train outflow at station *i* in direction 1 (pass/hr)

(7) = train outflow at station *i* in direction 2 (pass/hr)

Table 4 Results of Stage-I Optimization

Train Station (<i>i</i>)	Bus Route (<i>j</i>)	Bus Headway H_{ij} (hr)	BOUC (\$/hr)
1	1	0.236	3496.29
	2	0.307	
	3	0.330	
	4	0.157	
	5	0.149	
	6	0.565	
2	1	0.194	1656.52
	2	0.296	
	3	0.559	
	4	0.256	
5	1	0.184	3039.21
	2	0.285	
	3	0.349	
	4	0.196	
11	1	0.287	3376.55
	2	0.317	
	3	0.342	
	4	0.123	
	5	0.119	

BOUC= bus operator and user costs for bus routes connected at transfer station *i*
(Optimal train headway = 0.270 hours. Train operator and user cost = 1550.66 \$/hr.)

Table 5 Results of Stage-II Optimization

Iteration #	Coordinated Bus Group				Total Cost (\$/hr)	Common Headway (hr)
	Station 1	Station 2	Station 5	Station 11		
1	1,2,3,4,5,6	1,2,3,4	1,2,3,4	1,2,3,4,5	12296.3	0.322
2	1,2,3,4,5,6	1,2,3,4	1,2,3,4	1,2,3,4	12435.1	0.321
3	1,2,3,4,5,6	1,2,3,4	1,2,3,4	1,2,3	12239.2*	0.345
4	1,2,3,4,5	1,2,3,4	1,2,3,4	1,2,3	12243.8	0.339
5	1,2,3,4,5	1,2,3	1,2,3,4	1,2,3	12354.0	0.331
6	1,2,3,4,5	1,2,3	1,2,3	1,2,3	12547.0	0.326
7	1,2,3,4	1,2,3	1,2,3	1,2,3	12696.4	0.324
8	1,2,3,4	1,2	1,2,3	1,2,3	12764.9	0.322
9	1,2,3	1,2	1,2,3	1,2,3	12678.2	0.342
10	1,2,3	1	1,2,3	1,2,3	12672.0	0.330
11	1,2	1	1,2,3	1,2,3	12742.4	0.319
12	1,2	1	1,2	1,2,3	12808.5	0.306
13	1,2	1	1,2	1,2	12867.4	0.295
14	1	1	1,2	1,2	12922.1	0.284
15	1	-	1,2	1,2	12931.2	0.291
16	1	-	1	1,2	12985.4	0.282
17	1	-	-	1,2	12962.1	0.307
18	-	-	-	1,2	12992.2	0.315
19	-	-	-	1	13073.5	0.288

* Minimum total cost achieved by coordinated operation

Table 6 Benefits of Coordination and Optimal Results

Benefits of Coordination								
Cost Components	Stages	Costs (\$/hr)						
		Buses				Train	Total	
		Station 1	Station 2	Station 5	Station 11			
Wait	Stage 2	854.00	326.60	677.39	521.72	244.71	2624.42	
	Stage 1	535.93	262.78	448.29	489.37	191.38	1927.75	
	Savings	-318.07	-63.82	-229.10	-32.35	-53.33	-696.67	
Transfer	Stage 2	293.68	131.16	230.41	385.72	122.43	1163.40	
	Stage 1	552.54	309.95	447.28	529.70	340.29	2179.76	
	Savings	258.74	178.79	216.87	143.98	217.86	1016.36	
In-vehicle	Stage 2	1224.88	459.03	1153.20	1176.45	475.84	4489.40	
	Stage 1	1176.86	452.17	1122.84	1173.88	475.02	4400.77	
	Savings	-48.02	-6.86	-30.36	-2.57	-0.82	-88.63	
User	Stage 2	2372.56	916.79	2061.00	2083.89	842.98	8277.22	
	Stage 1	2265.33	1024.90	2018.42	2192.94	1006.70	8508.29	
	Savings	-107.23	108.14	-42.58	109.05	163.72	231.07	
Supplier	Stage 2	981.84	605.55	790.85	1156.30	427.47	3962.01	
	Stage 1	1230.96	631.62	1020.79	1183.61	543.97	4610.95	
	Savings	249.12	26.07	229.94	27.31	116.5	648.94	
Total	Stage 2	3354.40	1522.33	2851.85	3240.18	1270.45	12239.2	
	Stage 1	3496.29	1656.52	3039.21	3376.55	1550.66	13119.2	
	Savings	141.89	134.19	187.36	136.37	280.21	880.02	
Optimal Headways and Slack times								
Bus Route (j)	Headway (hr)				Slack Time (hr)			
	Station 1	Station 2	Station 5	Station 11	Station 1	Station 2	Station 5	Station 11
1	0.345	0.345	0.345	0.345	0.055	0.035	0.038	0.053
2	0.345	0.345	0.345	0.345	0.062	0.042	0.050	0.046
3	0.345	0.345	0.345	0.345	0.063	0.041	0.056	0.044
4	0.345	0.345	0.345	0.123	0.049	0.041	0.037	-
5	0.345	-	-	0.119	0.037	-	-	-
6	0.345	-	-	-	0.035	-	-	-

(Train headway = 0.345 hr)

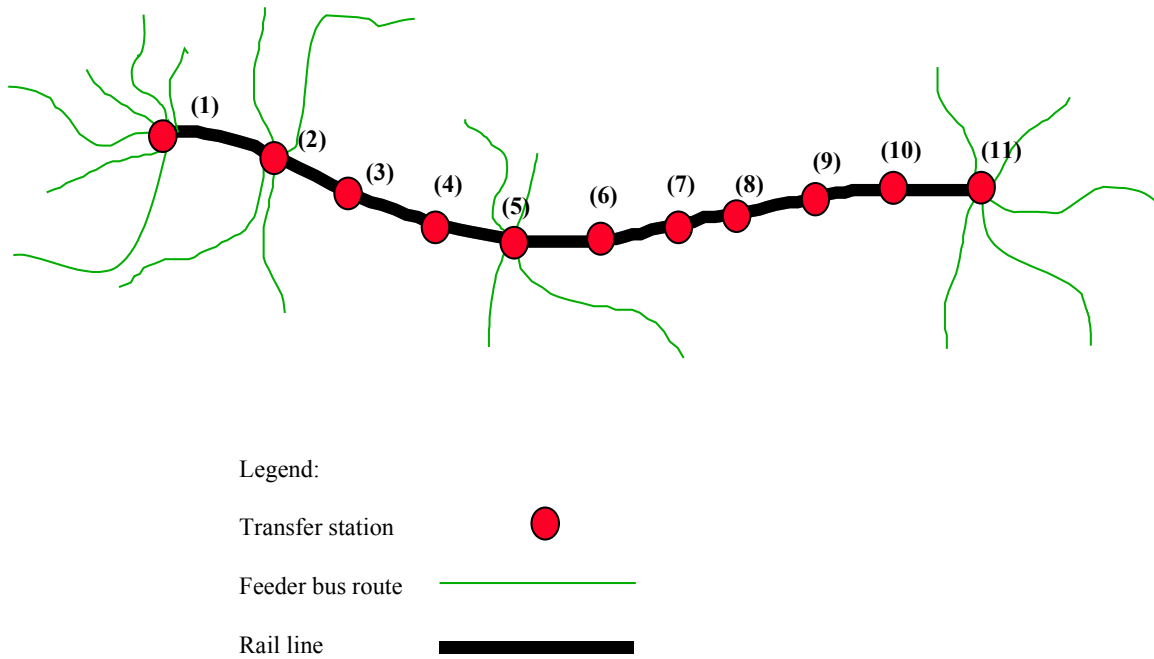


Figure 1 Configuration of the Intermodal Transit Network

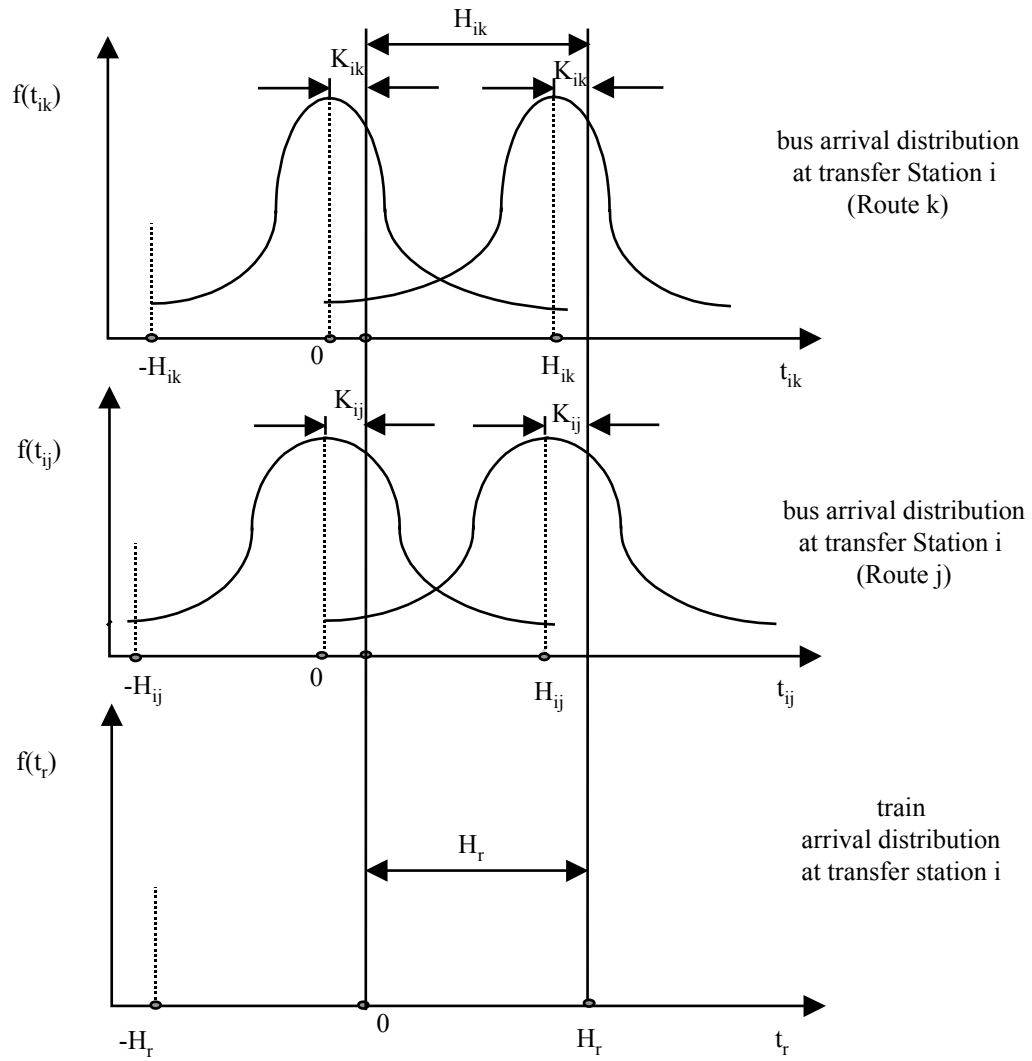


Figure 2 Bus and Train Arrival Distributions

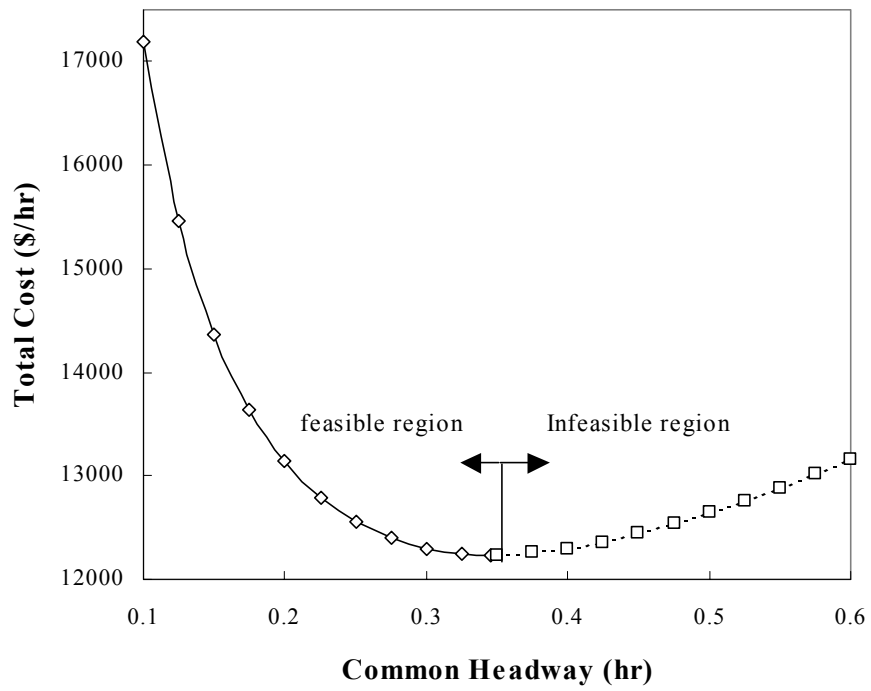


Figure 3 Total Cost vs. Common Headway

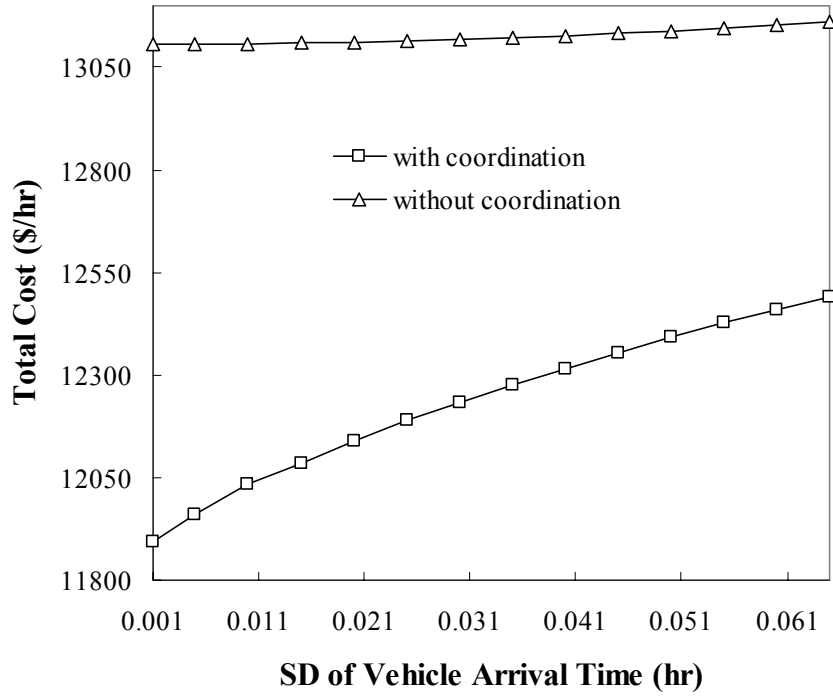


Figure 4 Standard Deviation (SD) of Vehicle Arrival Time (Route 1 to 6 at Station 1) vs. Total Cost